UNIT 11 STATISTICAL QUALITY CONTROL

Structure **Page No.**

11.1 INTRODUCTION

A widely accepted definition of the *quality* of a product is **its fitness for use** *for its intended purpose.* For example, a ball pen should write well throughout its life. Besides, the cap should not be loose, neither it should leak nor break easily, etc., are some of the other features. Again, for a cricket ball, some of the quality characteristics are like its *weight, size, shining, quality of stitches*, etc. And, for a water tap washer, these are its *thickness, inner diameter, outer diameter*, etc.

The *quality* of a product is assessed by the totality of its features. Have you ever thought of when and where products are made? We only curse the products (and the people who made them) whenever these products give us trouble or displeasure. Rarely we think how these products are made or what precautionary measures have been taken during the production process to ensure that the products are of *good quality.*

Quality Control (**QC**, in short) consists of procedures and methodologies which ensure that the quality characteristics of a product conform to its specifications.

Who is responsible for the quality of a product? Obviously, it is the manufacturer of the product. In this unit, we shall discuss some simple statistical tools, which are extensively used in the production process to ensure the quality of products. The most effective use of *Statistical Quality Control* (**SQC**, in short) generally requires cooperation among those responsible for the three different types of functions: *specification, production*, and *inspection.*

In Sec.11.2, you will get introduced to the concept of quality and learn about methods used in the process of controlling and systematically improving quality of a product. In Sec.11.3, we shall discuss the primary tools of *statistical process control* that are useful in monitering the quality aspects of a product. In Sec.11.4, you will learn about *acceptance sampling plans* - a technique used in ensuring that the produced products conform to the specified quality standards.

Objectives

After reading this unit, you should be able to

- explain the term *quality* and the phrase *statistical quality control*;
- describe the concept of *variation, chance* and *assignable cuases*;

- construct and interpret *control charts* for variables and attributes;
- estimate *process capability* from control charts data;
- describe *acceptance sampling plans*;
- interpret and use *oc curves* in determining *acceptable quality level, producer's risk, consumer's risk*, and *lot tolerance percentage defective*;
- describe *single sampling plan* and construct some simple single sampling plans with help of a *binomial nomograph*.

11.2 CONCEPT OF QUALITY

Everyday, from the time we get up and till we go back to bed, each one of us is busy with a number of activities. And, we depend on number of objects to carry out these activities. For example, we need a tooth brush, a tooth paste, wash basin or a water tap, the soaps and detergents, the stoves that we use for cooking, the vehicles that we use for going to our offices, electrical bulbs, phones, medicines and what not?

Have you ever noticed that there is a tiny ball in the nib of every ball pen?

Quality of any product depends upon certain **characteristics** related to the product and the materials that go into it. For example, in the case of a ball pen, if the diameter of the ball is undersized then the pen is bound to leak. Similarly, if the refill length is oversized, then the press button may not work properly. So, *ball diameter* and *refill length* are two **quality characteristics** for a simple product like ball pen.

In fact, a ball pen has many more quality characteristics. But, unlike diameter and length, not all quality characteristics are *measurable*. Recall, *non-measurable characteristics* are called **attributes** and the *measurable* ones are called the **variables**.

11.2.1 Nature of Quality Control

Let us continue our discussion with the example of a ball pen. As you may agree, certain brands of ball pens stop writing from the second or third day onwards; some write well only on certain types of papers; some write well on almost any kind of paper and write till the last drop of ink remains in the refill. When a pen writes well, we say it is of *good quality*.

How we decide the *quality* of a product? At the time when a product is designed, certain *specifications* or *levels of tolerances* are established on all important quality characteristics of the product. For example, in case of a ball pen, the specifications on refill length may be that it should lie between 9.80 cms and 10.20 cms. So, if we can ensure that all the quality characteristics of a product are maintained within their specified limits, then automatically the quality of that product will be good.

Cooking in hotels is an example of a manufacturing process.

But, we know that a *manufacturing process* is an interaction among people, equipments, materials, methods and environment, wherein output could be either another product or a component that goes into an end product. Thus, the performance of a *process* is indicated by the quality of its output and can be assessed by examining the *quality characteristics* of the output. So, a *process* is operated in such a way that the quality characteristics of output product is maintained at desired levels. This is called **controlling a process**.

In 1924, **Walter A. Shewhart** of Bell Telephone Laboratories introduced *statistical control charts* as a tool for controlling quality of industry products. From then onwards, people slowly started recognising the use of statistical techniques in quality control. Today, it is known world over that statistical techniques are not only indispensible for quality control but also play a very crucial role in all other facets of quality related matters.

Statistical Quality Control techniques can be broadly divided into two categories: (i) *Statistical Process Control* (**SPC**, in short) techniques; and (ii) *Acceptance Sampling*.

Statistical Quality Control

SPC techniques are widely used in almost any manufacturing process and are very useful in solving real situation problems, achieving *process stability*, and *making continuous improvements* in product quality. The most important among these are **control charts**. We shall discuss *control charts* in the next section.

SPC techniques have wide applications in non-manufacturing processes as well.

In many situations, however, one or more components of a product are bought from outside agents and the manufacturer does not have a direct control over the quality of the components. Then, in such cases, **acceptance sampling techniques** are useful in ensuring that the bought out components conform to specified quality levels.

In the next section, we would discuss *SPC techniques* and *acceptance sampling* will be discussed in Sec.11.4.

11.3 STATISTICAL PROCESS CONTROL

Statistical Process Control is a methodology used for understanding and monitoring a process by collecting the data on quality characteristics periodically from the process, analysing them and taking necessary actions based on the analysis results.

From now onwards, we will focus our discussion by referring to M/s BP & Company, a ball pen manufacturing company.

One of the sections in this company produces refills for the ball pens. The specifications on the refill length, one of the quality characteristics of refill, are 10 ± 0.2 cms. While producing the refills how does one ensure that their lengths conform to these specifications?

Refill length problem

11.3.1 Concept of Variation

As you may agree, however well the process is maintained, certain amount of variation in the lengths of the refills is unavoidable. But, if this process is operated under stable conditions i.e., *machine settings are same, quality of the materials used is same, operators are equally experienced*, etc., then the quality characteristics such as refill length normally exhibit a specific *patterns of variation.*

Pattern of variation means *statistical distribution.*

That is, if a process is operating under stable conditions then the amount of variation in the quality characteristic is usually small and is a result of several small causes. These small causes are known as *chance causes* and are usually unevitable.

The resulting variation is called the *chance cause variation* (or the *inherent variation*) of the process. In practice, we find that most processes are often disturbed inadvertantly or otherwise.

Chance Causes

On the other hand, a change in the machine settings, sudden drop in the quality of raw material or induction of a new operator due to an absence of the regular operator, etc., are some of the causes that might disturb a *stable process*. The reasons for variation outside this stable process may be discovered and corrected. These causes are known as *assignable causes.*

When the *assignable causes* are prevailing in a process, the process becomes unstable and this is reflected in the behaviour of the quality characteristic. That is, there will be frequent changes in the distribution of the quality characteristic. As a result, there will be more variation in the data. This variation is called the *variation due to assignable causes.*

Assignable Causes

It is important to remember that when a process is operating only under *chance causes*,

we say that the process is statistically stable or that *the process is under statistical control.*

The power of *Shewhart control charts* lies in its ability to separate out assignable causes of quality variation. So, a control chart is used to monitor the stability of a process and alert us as and when an assignable cause creeps into the process. Also, control charts are very useful in detecting gradual improvement or deterioration in a process.

11.3.2 Control Charts

On-line process control means monitoring a process by periodically examining sample outputs of the process and taking corrective actions as and when necessary.

The concept of *control charts* is one of the most powerful techniques for *on-line process control.* Besides monitoring, control charts are useful also in the evaluation of capability of a process and in making continuous improvements in the process.

Recall the *refill length problem* mentioned above. We can use control chart technique to effectively solve this problem. But, firstly, let us see how a control chart is constructed.

Typically, a control chart is a two-dimensional graph in which x-*axis represents the* **sample numbers** and y-*axis represents a* **quality characteristic**. It has a solid *center line* (**CL**) and two dotted lines called *upper control limit* (**UCL**) and *lower control limit* (**LCL**) (see Fig. 1).

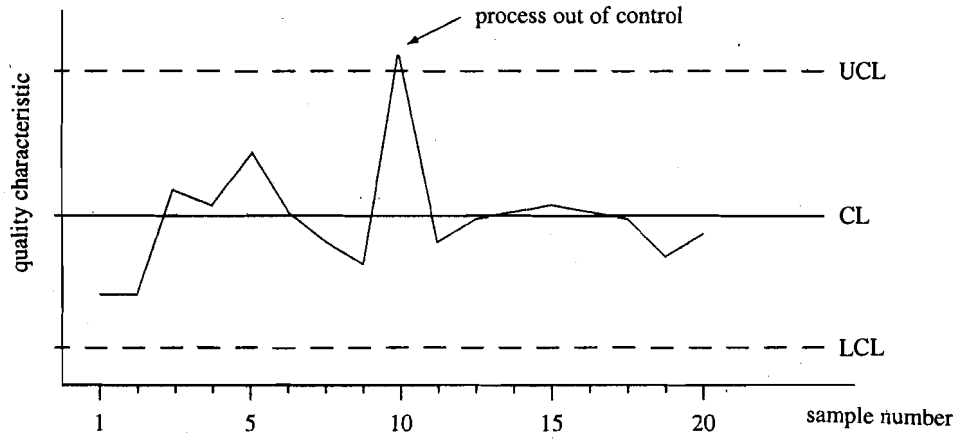

Fig. 1

Thus, the **construction of control charts** involves *collecting samples periodically from the process, computing the quality characteristic for each sample and plotting it against the sample number. The consecutive points are joined by line segments.*

As long as the plotted points are within the *upper* and *lower control limits* and do not exhibit any specific patterns, we have no evidence that the process is *not under statistical control.* When a point falls outside the control limits (below LCL or above UCL), it is a cause and indicates the presence of an assignable cause with a high probability.

However, *control chart* cannot tell us what went wrong with a process when something has gone wrong. It will only indicate that possibly something has gone wrong with the process. In fact, it is the responsibility of supervisor or QC manager to find out what has gone wrong.

Three Sigma Limits

In most situations, a *quality characteristic follows a normal distribution* or can be approximated by a normal distribution. Also, we know that the probability of a normally distributed random variable taking values below $\mu - 3\sigma$ or above $\mu + 3\sigma$, where μ is the mean and σ is the standard deviation, is very low (equal to 0.0027).

Therefore, if an observation falls outside **3σ limits**, it is logical to suspect that possibly something might have gone wrong. For this reason, *the control limits on a control chart are set up using 3σ limits.* Consequently, when a point falls outside the control limits on a control chart, it is more likely that it is due to the presence of an assignable cause,

rather than a mere chance cause.

Depending on the nature of quality characteristics, control charts are divided into two categories: (i) *control charts for variables*; and (ii) *control charts for attributes*.

Before we proceed further with our discussion, try the following exercise.

E1) For what type of quality characteristic do you think is a control charts for variables suitable? Cite some quality characteristics that need control charts for attributes.

Control charts for variables are adopted in situations where the quality characteristic is of measurable type. In the next part of the section, we shall discuss with you $\bar{x}$ - R charts – a type of control charts for variables.

11.3.3 Control Charts For Variables

Once again we shall refer to the refill length problem to explain these charts. Consider the data on refill length as given in Table 1. Observe that the samples are taken at 30-minute intervals. Here, each sample corresponds to lengths of *five refills* produced at the time of collecting the sample. Each sample is called a **subgroup**.

Table 1 : Refill length data

S.No.	Date	Time	Subgroup 1	2	3	4	5	Average	Range
1	23.12.97	08:00	10.11	10.08	10.14	10.10	10.15	10.116	0.07
2		08:30	10.08	10.08	10.12	10.13	10.06	10.094	0.07
3		09:00	10.07	10.22	10.01	10.11	10.07	10.096	0.21
4		09:30	10.21	10.10	10.09	10.13	10.02	10.110	0.19
5		10:00	10.12	9.98	9.91	10.05	10.17	10.046	0.26
6		10:30	10.17	10.14	10.08	10.06	10.23	10.136	0.17
7		11:00	10.10	10.11	10.21	10.05	10.22	10.138	0.17
8		11:30	10.10	10.06	10.23	10.14	9.97	10.100	0.26
9		12:00	10.10	9.96	10.13	10.14	10.04	10.074	0.18
10		12:30	10.05	10.19	10.13	10.10	10.08	10.110	0.14
11	24.12.97	08:00	10.08	10.05	10.05	10.08	10.16	10.084	0.11
12		08:30	9.91	10.21	10.00	10.02	10.29	10.086	0.38
13		09:00	10.11	9.98	9.97	10.04	10.08	10.036	0.14
14		09:30	10.08	10.21	10.13	10.16	10.04	10.124	0.17
15		10:00	9.99	10.14	9.96	10.09	10.07	10.050	0.18
16		10:30	10.17	10.18	10.04	9.99	10.11	10.098	0.19
17		11:00	10.06	9.92	10.10	10.06	10.02	10.032	0.18
18		11:30	10.16	10.12	10.16	10.02	10.19	10.130	0.17
19		12:00	10.14	10.04	10.14	10.02	10.07	10.082	0.12
20		12:30	10.08	10.07	9.97	10.09	10.12	10.066	0.15

Thus, in above table, we have data for 20 *subgroups*. The variation in the five observations of any subgroup can be attributed only to *chance causes* because the *five observations* correspond to *five refills* that were produced almost at the same time and it is very unlikely that they were affected by any assignable causes in such a short span of time.

Next, let us talk about the *frequency of sampling*. Supposing the samples are taken once in every five minutes instead of every 30 minutes, then we are not going to find big differences or changes in consecutive subgroups. So, *too frequent sampling is an unnecessary labour.*

While keeping these points in mind, one has to decide the *selection of subgroups* and *their frequency* in such a way that the variation in observations within a subgroup is only due to chance causes and the variation among subgroups is likely to be affected by assignable causes. The subgroups selected in this way are called the **rational subgroups.**

Recall that the refill lenght specifications are 10 ± 0.2 cms. So, we will be too happy if the refills are produced with length exactly equal to 10 cms. But we know that this is practically impossible. So, we try to ensure that atleast the average refill length is as close to 10 cms as possible and the variation in the refill length as low as possible.

Here, **LSL** and **USL** stands for the *lower* and *upper specification limits*, respectively.

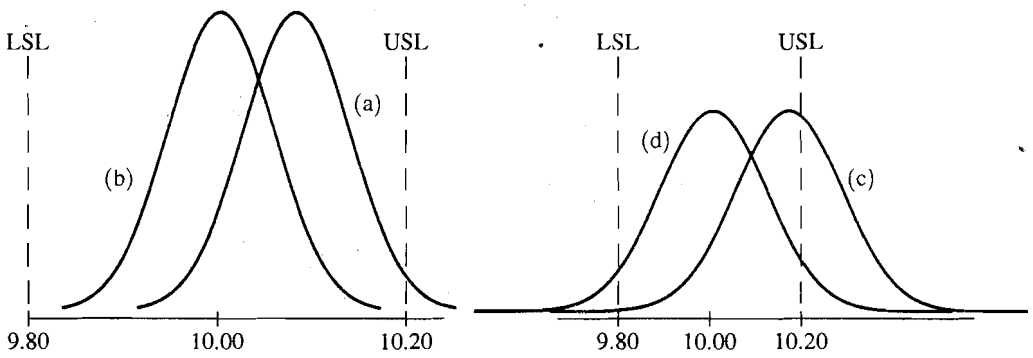

Fig. 2

The four curves (a – d) in Fig. 2 describe four different distributions for refill lengths. Observe that, for the curve (a), the mean length is high even though the variation is low. So, if our process produces refills like this, then some percentage of the refills do not conform to the length specifications. However, if we can adjust the process so that the mean length is equal to 10 cms, then our process will be good. Then, in this situation, the refill length distribution will look like the one in (b).

On the other hand, if the distribution is given by (c) then even after adjusting the mean to the target there will be *non-conformance* with regard to length (see curve (d)). So, in this case, we must improve the process to reduce the variation.

Controlling Mean and Variability

The moral of the story is that, *when the quality characteristic is a variable, it is essential to control both mean and variation.* For this reason, *two separate control charts* – one *for controlling the mean* and the other *for controling variability*, are maintained for a measurable quality characteristic.

In quality control terminology, when a process is stable, the mean of a quality characteristic under study is referred to as **process mean** and its variability is referred to as **process variability**.

Here, *we denote the process mean by μ and process standard deviation by σ.*

$\bar{x}$ - R Charts

Now, let $x_1, x_2, \ldots, x_5$ be five independent observations of a *subgroup* on the refill length. If the subgroup is *rational*, we can expect that $x_1, x_2, \ldots, x_5$ are random variables having same mean and standard deviation.

Recall that range is the difference between the largest and smallest values of the observations.

Let $\bar{x}$ and R denote the *average* and *range*, respectively, of these five observations. Then, $\bar{x}$ is called the **subgroup average** and R the **subgroup range**.

Two separate charts are maintained in an $\bar{x}$ - R chart: (i) $\bar{x}$ - *chart*; and (ii) R - *chart*. In the $\bar{x}$ - chart, used for controlling the *process mean*, we plot the *sample averages against the sample numbers*; and, in the R -chart, used for controlling *process variability*, we plot *the sample range against the sample number.*

Recall, the *control limits* are worked out based on the 3σ-limits concept. Also, from what your read in Unit 4, we know *if* $x_1, x_2, \ldots, x_n$ *are independent random variables with mean μ and standard deviation σ, then $\bar{x}$ has mean μ and its standard deviation is equal to* $\frac{\sigma}{\sqrt{n}}$ (by *central limit theorem*).

As such, since we plot the sample averages on the $\bar{x}$ - chart, we should construct the *center line* and *control limits* using the mean and standard deviation of the averages and not of the individual observations. Thus, the CL, LCL and UCL for the $\bar{x}$ - chart are

given by

$$CL = \mu,\ LCL = \mu - 3\frac{\sigma}{\sqrt{n}} \text{ and } UCL = \mu + 3\frac{\sigma}{\sqrt{n}}, \quad (1)$$

where n is the sample size (For example, n = 5 for the refill length problem). And, the estimates of μ and σ are given by $\hat{\mu} = \bar{\bar{x}}$ and $\hat{\sigma} = \bar{R}/d_2$, where $\bar{\bar{x}}$ is the average of all subgroup averages, $\bar{R}$ is the average of subgroup ranges, and d_2 is a constant depending on n. Substituting these estimates in Eqn.(1), we get

In practice, we do not know the values of μ and σ. So, we estimate these values from the data and replace μ and σ by their estimates in Eqn.(1).

$$CL = \bar{\bar{x}},\ LCL = \bar{\bar{x}} - 3\frac{\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} - A_2\bar{R},\ UCL = \mu + 3\frac{\bar{R}}{d_2\sqrt{n}} = \bar{\bar{x}} + A_2\bar{R}, \quad (2)$$

where $A_2 = \dfrac{3}{d_2\sqrt{n}}$.

Similarly, the control limits for an R - chart are also worked out based on the 3σ limits concept and these are given by

$$CL = \bar{R},\ LCL = d_3\bar{R} \text{ and } UCL = d_4\bar{R}, \quad (3)$$

where d_3 and d_4 are constants depending n. In our subsequent discussion, we shall make use of the values of d_2, d_3, d_4 and A_2 as given in Table 2.

Table 2 : *Control chart constants*

Sample size, n	d_2	A_2	d_3	d_4
2	1.128	1.88	0	3.27
3	1.693	1.02	0	2.57
4	2.059	0.73	0	2.28
5	2.326	0.58	0	2.11
6	2.534	0.48	0	2.00
7	2.704	0.42	0.08	1.92

For example, when n = 5, $d_2 = 2.326$, $A_2 = 0.58$, $d_3 = 0$ and $d_4 = 2.11$. Let us use these values to solve the following problem.

Problem 1. Estimate μ and σ for refill length data and work out the control limits for $\bar{x}$ and R - charts.

Solution. Using data given in Table 1, we get

$$\bar{\bar{x}} = \frac{10.116 + 10.094 + \ldots + 10.066}{20} = \frac{201.808}{20} = 10.09, \text{ and}$$

$$\bar{R} = \frac{0.07 + 0.07 + \ldots + 0.15}{20} = \frac{3.51}{20} = 0.175.$$

Then, the estimates for μ and σ are given by

$$\hat{\mu} = 10.09 \text{ and } \hat{\sigma} = \frac{0.175}{2.326} = 0.0752.$$

Therefore, control limits for $\bar{x}$ - chart are given by

$$CL = \bar{\bar{x}} = 10.09;\ LCL = \bar{\bar{x}} - A_2\bar{R} = 10.09 - 0.58 \times 0.175 = 9.99; \text{ and}$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 10.09 + 0.58 \times 0.175 = 10.19.$$

Similarly, the control limits for R - chart are given by

$$CL = \bar{R} = 0.175;\ LCL = d_3\bar{R} = 0.0 \times 0.175 = 0.0; \text{ and}$$

$$UCL = d_4\bar{R} = 2.11 \times 0.175 = 0.369.$$

At this stage, we may call these control limits as **trial control limits** because we do not know whether the data we have analysed correspond to a *stable process* or not.

Now, you try the following exercise.

E2) A cricket ball manufacturing company wants to maintain control charts for the weight of the balls. Twenty-five samples, each of size 4, were collected. The *sum of sample averages* and the *sum of sample ranges* were found to be 7575 grams and 154 grams, respectively. Estimate the process mean and standard deviation, and compute the control limits for $\bar{x}$ and R - charts.

Using data from Table 1, $\bar{x}$ and R - charts for Problem 1 are as shown in Fig. 3.

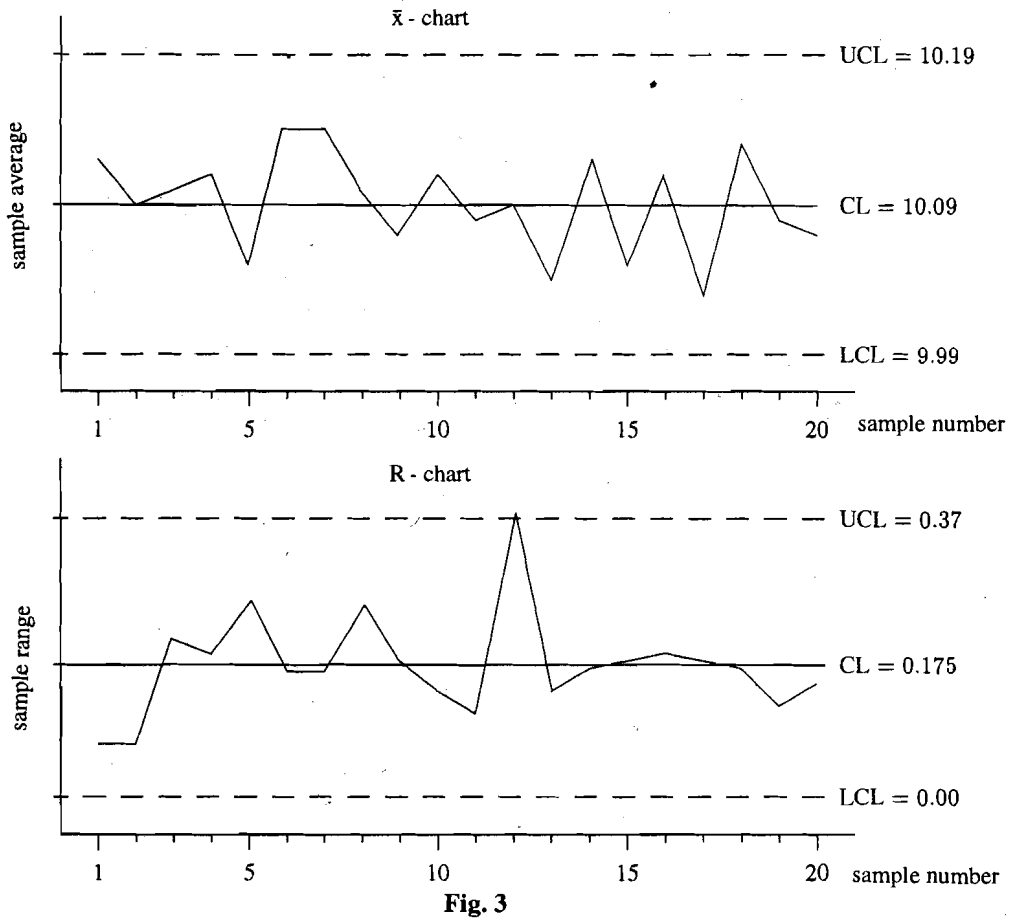

Fig. 3

Note that the 12th point on the R - chart indicates an out of control situation with regard to variability. The 12th subgroup might not be similar to the other subgroups, indicating the process might not be stable.

Though the actual cause could not be ascertained, there must have been one or more *assignable causes* which have caused this high variation. In this situation, it is better to recalculate the control limits omitting the susceptible subgroup number 12.

For this reason, we called the earlier limits, *trial control limits.*

You try to do that in the following exercise.

E3) Discard the data of 12th subgroup in Table 1 and estimate μ and σ. Compute the control limits for $\bar{x}$ and R - charts with the remaining 19 subgroups. Do the data indicate statistical control without 12th subgroup?

The process of recalculating the control limits after eliminating the *outlying* subgroups is called **homogenization**.

While *homogenizing*, if we have to discard more than 25% of the subgroups, then it is better to discard the entire data and collect fresh data. But, while collecting data for control chart analysis, it is important to ensure that *process conditions remain the same throughout the data collection period.*

Now, from the solution of E3, we know that if we redraw the control charts after eliminating the 12th subgroup then both $\bar{x}$ and R- charts exhibit a state of statistical control. But then, does it mean that the process meets the requirements?

Problem 2. Do the refills produced by this process conform to their length specifications?

Solution. Let X stands for the length and we assume that it is distributed normally with the mean and standard deviation as 10.09 and 0.075, respectively. Then, we find that

$$P[X > 10.20] = P\left[\frac{X - 10.09}{0.075} > \frac{10.20 - 10.09}{0.075}\right] = P[Z > 1.47] = 0.07,$$

where Z is the *standard normal distribution.* This means about 7% of the refills produced by the process are oversize. It is clear from the $\bar{x}$ - chart that the process mean

is *set* at 10.09 cms (see Fig. 3).

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After observing this, the QC manager suspected that there was a setting problem in the refill cutting machine. But, before carrying out any investigation, he decided to analyse the variation aspect too.

11.3.4 Process Capability Analysis

Supposing that QC manager has corrected the process so that the mean is as desired, do you think that the refills will conform to their length specifications? The answer is **NO**.

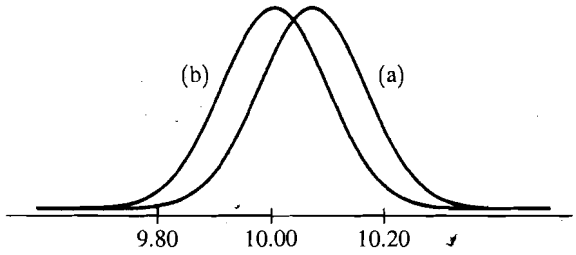

Fig. 4

In Fig. 4, curve (a) represents the process before adjusting for the mean and (b) represents the process after adjusting the mean. Note that there is no change in the variability. Thus, we will continue to have rejections even after adjusting for mean unless the process variability itself is reduced.

At this stage, we have to answer two question: (1) What is the existing variability? (2) How much should we reduce it by? To answer these questions, we need the following definitions.

Definition. *Total tolerance* of a measurable quality characteristic, denoted by $\mathbf{T}$, is given by the difference $\mathbf{T = USL - LSL}$, where USL and LSL are the *upper* and *lower specification limits*, respectively.

For example, as LSL = 9.80 cms and USL = 10.20 cms for refill length problem, so, the *total tolerance* T = 0.4 cms in this case.

Definition. When a process is under statistical control, its *process capability* is given by $\mathbf{6\sigma}$, where σ is the *process standard deviation*.

The concept of 3σ-limits is used to define the **process capability**.

The first question that we raised above can be answered by specifying an estimate of the process capability. Here, in our situation,

$$\textit{estimate of the process capability} = \mathbf{6\overline{R}/d_2}.$$

For refill length problem, the specification limits are 10 ± 0.2 cms. So, to avoid rejections, we must necessarily have a process for which the 3σ limits lie within the specification limits after setting the mean at the target (see Fig. 5).

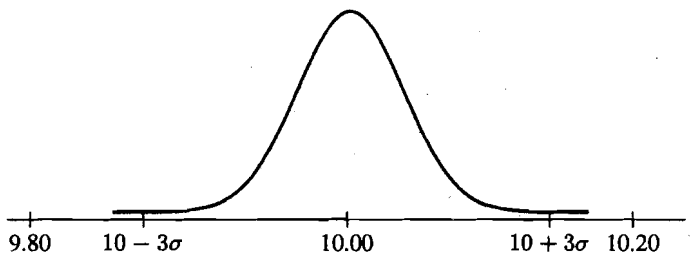

Fig. 5

In other words, the *process capability must be* **less than** *the total tolerance*. Therefore, the answer to the second question we raised above is that the *process capability should not exceed the total tolerance*.

Definition. The *process capability ratio* of a stable process is the ratio of total tolerance to the process capability and is denoted by C_p. That is,

$$C_p = \frac{\text{total tolerance}}{\text{process capability}} = \frac{USL - LSL}{6\sigma}.$$

Thus, by what we have said above, if $C_p < 1$, then the process is bound to produce rejections even when the mean is set on target. And, if $C_p \geq 1$, the rejection percentage will be almost zero, provided the mean is at the target.

An estimate of C_p can be obtained by substituting the estimate for σ in the above formula. For example, an estimate of C_p for the refill length problem under study is given by

$$\hat{C_p} = \frac{10.20 - 9.80}{6 \times 0.075} = 0.89.$$

Since the estimate of C_p is less than 1, we may infer that the *refill length process is not capable*. More generally, even if we get the estimate of C_p slightly more than 1, we may still consider the process incapable. This is because our estimate for C_p may be an underestimate due to sampling fluctuations.

Precisely for this reason, a process is considered to be capable, only if the estimate of C_p is atleast 1.33.

Try the following exercise now.

E4) Give an example of
   a) a process whose C_p = 1.5 but has high rejections on USL side; and
   b) a process whose C_p = 1.5 but has high rejections on LSL side.
   (*Hint:* Specify the process parameter μ and σ.)

With above analysis in hand, the QC manager carried out an investigation of the process and found out that a unit of the machine used for setting the refill length was not properly calibrated. He also found that certain parts in the machine were worn out which were causing vibrations in the length cutting machine.

Subsequently, he got the unit recalibrated and replaced the worn out parts and collected five samples from the process. The new data is given in Table 3.

Table 3 : Refill length data after corrective action.

S.No.	Date	Time	Subgroup 1	2	3	4	5	Average	Range
21	6.1.98	08:00	9.88	10.02	9.94	9.86	10.04	9.925	0.18
22		08:30	9.99	10.08	10.03	10.01	10.04	10.028	0.09
23		09:00	10.00	10.06	9.98	10.03	10.01	10.018	0.08
24		09:30	9.94	9.92	9.95	10.00	10.02	9.953	0.10
25		10:00	10.03	10.05	10.08	10.08	10.09	10.060	0.06

The new points are plotted on the charts shown in Fig. 1 and new charts now look like as in Fig. 6.

Note the clear distinction between the behaviours of the first 20 points and the last 5 points on both $\bar{x}$ and R - charts. The difference is the effect of a change in the process after the first 20 points. And the change is the result of the corrective actions taken by the manager.

Since some changes were made in the process, we must reconstruct the control charts with new data. To complete the task, manager has collected 15 more samples. The *sample averages* and *ranges* are summarised in Table 4.

Table 4 : Summary of 15 more samples on refill length data

S.No.	Average	Range	S.No.	Average	Range	S.No.	Average	Range
26	10.035	0.14	31	10.053	0.08	36	10.048	0.18
27	9.995	0.14	32	9.973	0.08	37	10.030	0.14
28	10.020	0.08	33	9.983	0.20	38	10.048	0.22
29	9.970	0.13	34	10.020	0.16	39	10.073	0.15
30	9.970	0.10	35	9.990	0.11	40	9.985	0.23
Total	49.99	0.59	Total	50.019	0.63	Total	50.184	0.92

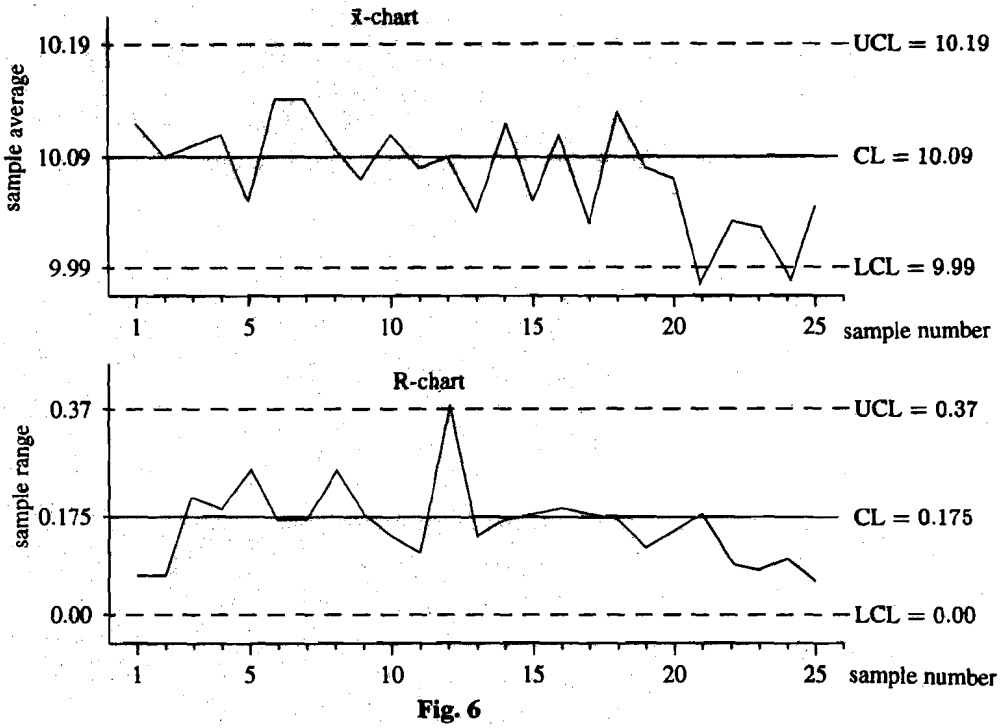

Fig. 6

You can help and advise the manager with the above data, provided you do the following exercise.

E5) Combining the information provided in Table 3 and Table 4, do the following.
- (a) Estimate the new process mean and standard deviation;
- (b) Construct the control charts;
- (c) State whether the process is under statistical control;
- (d) Estimate the process capability and process capability ratio; and
- (e) State your advice to the manager.

Next, let us talk about some control charts used for *non-measurable* quality characteristics.

11.3.5 Control Charts for Attributes

When products are inspected, they are classified into good and defective products. A defective product is one that has one or more defects. The performance of a process is often assessed by the *proportion of defective items* produced by the process or by counting the *number of defects per unit* of the product.

A defect is a *nonconformity* with respect to any of the quality characteristics of the product.

Control charts used in these situations are known as **attribute control charts**. Here, we discuss the following four such type of charts.

i) **p** *and* **np** *charts for the control of defective products;*

ii) **c** *charts for the control of number of defects per unit.*

p-charts

In case of BP & Co., 100 refills are selected at random from the process each day and are inspected for all quality characteristics. Based on the inspection results, each refill is classified as *good* or *defective*.

So, each day's sample of 100 refills is taken as a subgroup. The results of 14 days sample collection are given in Table 5. Here, X denotes the number of defective refills out of 100 inspected each day.

Table 5 : Refill inspection data

S. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Date	15	16	17	18	19	20	22	23	24	26	27	29	30	31	Total
X	9	5	6	7	6	5	6	8	7	4	6	7	6	2	84

When the process is stable, it is reasonable to *assume that (i) the probability of any refill being defective is same for all refills, and (ii) the event that any of the refills being good or defective does not influence the quality of other refills.*

Let

$$X_i = \begin{cases} 1, & \text{if the } i^{th} \text{ refill inspected is defective} \\ 0, & \text{otherwise} \end{cases} \qquad (i = 1, 2, \ldots, n),$$

where n is the sample size. For example, n = 100 in our situation. Then, the total number of defective refills in the sample is given by $X = X_1 + X_2 + \ldots + X_n$.

Under the assumptions (i) and (ii) above, X has a binomial distribution with parameters n and p, where p *is the common probability of any refill being defective*. Also, we know that the mean and standard deviation (SD) of a binomial distribution with parameters n and p are given by

$$\text{Mean}(X) = np \text{ and } SD(X) = \sqrt{np(1-p)}. \tag{4}$$

If y denotes the *proportion of defective items* in a sample of n items, then $y = \frac{X}{n}$.

In order to control the *proportion of defective items* produced by a process, we use a **p-chart**. In a p-chart, we plot the y *values against the corresponding sample numbers*. Unlike $\bar{x}$ - R charts, attribute control charts have only one chart to be plotted.To arrive at the control limits, we need the mean and standard deviation of y. These are given by

$$\text{Mean}(y) = \frac{\text{Mean}(X)}{n} = p \text{ and } SD(y) = \frac{SD(X)}{\sqrt{n}} = \sqrt{\frac{p(1-p)}{n}}. \tag{5}$$

Recall, p is the *proportion of defective items* produced by the process. In quality control terminology, p is referred to as the **process average**. An estimate of p is given by

$$\bar{p} = \frac{\text{total number of defective items in all the samples}}{\text{total number of items inspected}}.$$

Thus, if we have m samples, then

$$\bar{p} = \frac{\sum_{i=1}^{m} d_i}{\sum_{i=1}^{m} n_i}, \tag{6}$$

where $\mathbf{d_i}$ is the *number of defective items in the* i^{th} *sample* and $\mathbf{n_i}$ is the i^{th} *sample size.*

As before, the control limits in an attribute control chart are based on the 3σ *limits concept*. When all the subgroups are of same size (= n, say), then the control limits for a p-chart are given by

$$CL = p, LCL = p - 3\sqrt{\frac{p(1-p)}{n}} \text{ and } UCL = p + 3\sqrt{\frac{p(1-p)}{n}}. \tag{7}$$

Since we do not know the value of p, the control limits will be obtained by replacing p by its estimate $\bar{p}$ given by Eqn.(6). Thus, the control limits for a p-chart are given by

$$CL = \bar{p}, LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \text{ and } UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}. \tag{8}$$

Using data from Table 5, the control limits for the refill length problem are given by

$$CL = \bar{p} = \frac{9 + 5 + \ldots + 2}{14 \times 100} = \frac{84}{14 \times 100} = 0.06, \tag{9}$$

$$LCL = 0.06 - 3\sqrt{\frac{0.06(1-0.06)}{100}} = -0.011, \text{ and}$$

$$UCL = 0.06 + 3\sqrt{\frac{0.06(1-0.06)}{100}} = 0.131.$$

When the lower control limit happens to be negative, the control line is set at zero. So, LCL = 0 in the p-*chart* of refill length problem (see Fig. 7).

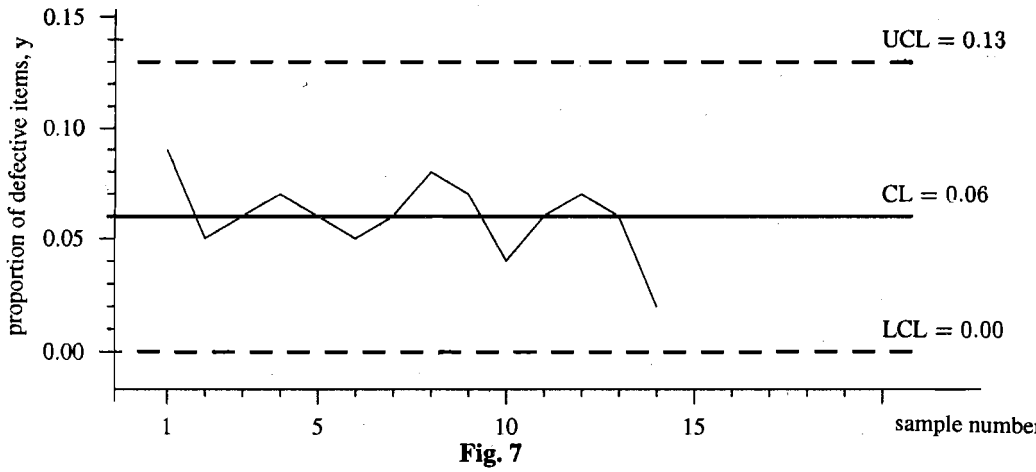

Fig. 7

We have to be cautious in interpreting a p-chart. The points which fall above the UCL are called the **high spots** and the points that fall below the LCL are called the **low spots**.

A *high spot* may be due to deterioration of the process or it could be due to a change in the inspection standard (more stringent inspection may result in larger number of defective items). Similarly, a *low spot* may indicate an improvement in the process or a deterioration in the inspection standards.

Try the following exercise.

E6) Let the refill inspection data collected be as given in the following table.

Recall, X denotes the *number of defective refills* out of 100 inspected each day.

Table 6 : Refill inspection data

S.No	1	2	3	4	5	6	7	8	9	10	
Date	6	7	8	9	10	12	13	14	15	16	Total
X	1	3	2	2	1	0	5	1	1	3	19

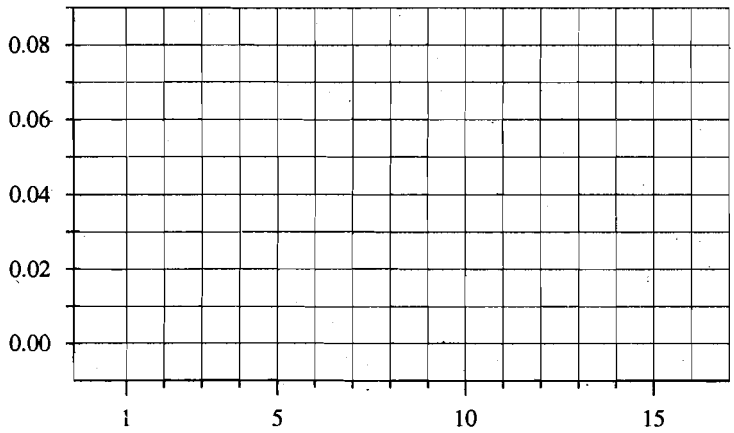

Fig. 8

(a) Plot the first five samples in Fig. 7 itself (use a pencil).
(b) Do you notice any change in the process? If so, what is the difference?
(c) Can you say, from the chart you plotted, that the process is out of control?
(d) Construct a new p-chart using only the data given in Table 6 and comment on the process (use the blank chart given in Fig. 8 above).
(e) Was there any improvement in the process? Estimate the rejection percentages for the two periods.

Varying Sample Sizes

Now, suppose we are not inspecting the same number of refills each day. Then, in this case, the subgroup sample sizes are varying. In such cases, the control limits will vary depending upon the subgroup sample size. Let us say, we have m subgroups with n_i sample items for the *ith* subgroup, then the control limits for the subgroups are given by

$$LCL_i = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}} \text{ and } UCL_i = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}. \quad (10)$$

The CL is drawn at $\bar{p}$ and there is no change in the formula for $\bar{p}$ (Eqn.(6) takes care of the unequal sample sizes).

You try the following exercise now.

E7) Construct the control limits of p-chart for the following data.

Table 7 : Data for p-chart with unequal sample sizes

S. No.	1	2	3	4	5
sample size	100	121	81	100	121
number of defective pens	2	2	0	1	2

Draw a rough sketch of the p-chart for this data.

---

np - charts

Sometimes it is necessary (or convenient) to look at the *number of defective items* rather than the *proportion of defective items*. In such situations, we use **np** - charts instead of **p** - charts. The only difference between p - *charts* and np - *charts* is that in the later case y-*axis represents the number of defective items in a subgroup.*

Try to convince yourself that the control limits on an np-chart should be

$$CL = n\bar{p},\ LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})},\ \text{and}\ UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})},$$

where $\bar{p}$ is same as defined for p-charts above.

Try the following exercise now.

E8) If the subgroup sample sizes are not the same, will the control limits vary in an np-chart? In particular, what can you say about the center line, CL?

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c - chart

Finally, let us discuss the *control charts, which are used to find the number of defects per unit*. These charts are useful in situations when the performance of a process is assessed by number of defects per unit. Note that a unit may be a single product or a fixed number of products.

PCBs are used in many electronic products such as TV, Computer, etc.

For example, a printed circuit board (PCB) having several hundreds of circuits built in it, may be treated as a unit. On the other hand, a bunch of ball pens packed in a carton may also be treated as a unit.

The quality characteristic plotted on a c-*chart* is the total number of defects per unit, denoted by **c**. Usually, Poisson distribution is a good approximation for **c**. Therefore, a c-*chart* is constructed based on the assumption that c follows Poisson distribution. We know that if m is the mean of a Poisson distribution, then its standard deviation is equal to $\sqrt{m}$.

Hence, the *control limits for a* c-*chart* are given by

$$CL = m, LCL = m - 3\sqrt{m},\ \text{and}\ UCL = m + 3\sqrt{m}.$$

As in the case of p-*charts*, the control limits are obtained by replacing m by its estimate $\bar{m}$ in the above formulae. If k is the total number of units inspected, then the average number of defects per unit, denoted by $\bar{m}$, is given by

$$\bar{m} = \frac{\text{total number of defects in k units}}{k}.$$

The following problem will explain what a c-*chart* is and how it is applied.

Problem 3. The assembly section of M/s BP & Co. has five groups of operators. Each group consists of 3 operators. The job of the groups is to assemble various components into ball pens and pack them in cartons. The groups are also responsible for identifying and setting aside the defective components while assembling. Each carton consists of 200 pens. A sample of one carton from each group is selected at random and all the pens in the carton are inspected. The total number of defects per carton (**c**) is recorded

for each group. The performance of each group is monitored by maintaining a *c-chart* for each group separately. For one of the group, find (i) the *average number of defects per unit*, (ii) the *control limits for c-chart*, and (iii) *plot the control chart.*

Solution. We plot the c-*chart* for group A of BP & Co. using the data as given in Table 8.

Table 8 : Assembly defects data for group A.

S. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
date	6	7	8	9	10	12	13	14	15	16	17	18	19	20	22	Total
c	3	3	3	5	4	4	6	1	10	4	11	7	3	5	3	72

Here, the *average number of defects per carton* is given by

$$\overline{m} = \frac{3+3+\ldots+5+3}{15} = \frac{72}{15} = 4.8.$$

And, the *control limits for c-chart* are given by

$$CL = 14.8,\ LCL = 4.8 - 3\sqrt{4.8} = -1.77 \text{ and } UCL = 4.8 + 3\sqrt{4.8} = 11.37.$$

Since we are plotting the *number of defects per carton* (c) on the chart, the control limits are drawn using LCL = 0 and UCL = 11.37. The corresponding control chart is as shown in Fig. 9.

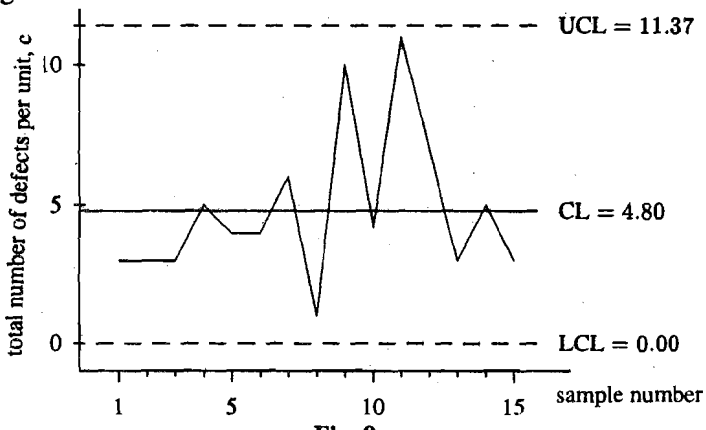


Fig. 9

Try the following exercise to have a comparison between groups A and E.

E9) The table below gives the defects for group E for the same period.

Table 9 : Assembly defects data for group E

S. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
Date	6	7	8	9	10	12	13	14	15	16	17	18	19	20	22	Total
c	9	10	12	13	12	8	10	12	14	2	8	10	11	10	9	150

(a) Estimate the average number of defects per carton.
(b) Is the assembly process of group E under statistical control?
(c) Which of the groups A and E, do you think is better?
(d) Explain how displaying the c-charts in front of these groups will help in improving the process.

In this section, we have discussed use of control charts in building quality into a product while it is being produced. Next, we discuss with you the concept of *acceptance sampling* – a techniques used to ensure that the produced products conform to specified quality standards. Here, it is important to remember that process controls are used to control and systematically improve quality, but acceptance sampling is not.

11.4 ACCEPTANCE SAMPLING

You know that, in most situations, we don't buy products directly from their

manufacturer. The products from the manufacturer are first supplied to dealers, the dealers supply them to retail shops and we buy them from retail shops.

Let us consider the case of Mr. Anil who is one of the dealers for M/s BP & Company. Mr. Anil buys ball pens from the company in large quantities. He receives the pens in lots where each lot contains 1000 pens. Mr. Anil will be too happy if all the lots that he receives have no defective pens at all. But, we know that this never happens in practice.

Inspection is the process of comparing actual measurable characterstics with pre-determined standard characteristics. And, 100% inspection means inspecting all items in the lot.

Can we then think of a procedure that will ensure that no lot has a defective pen? If so, at what cost? Is it worth adopting such a procedure? You might think that 100% inspection before packing is a procedure that ensures *defect-free* lots. In fact, 100% inspection is not always 100% efficient.

In some cases, there may be slips in inspection due to fatigue or measuring equipment errors. And, in some other situations, it may not be feasible to carry out 100% inspection. For example, if the product is bullets and inspection involves firing the bullets, then 100% inspection means, we will be left with nothing. So, what is the alternative?

We use sampling techniques in most such situations. In fact, sampling is extensively used in our day to day life. For example, we use sampling while purchasing vegetables and groceries, for selection of our family doctor, and so on.

A **sampling procedure** used to accept or reject a lot of items is known as an *acceptance sampling plan* (**ASP**, in short). Indeed, we use *acceptance sampling plan* as an audit tool to ensure that the product of a process conforms to requirements.

11.4.1 Sampling Plan Concepts

For a smooth discussion of the topic, we need an understanding of terms defined in the following definition.

It is assumed that there are no *inspection errors* i.e., an item, on inspection, found to be defective if and only if it is actually so.

Definition. *A* **lot** *is a collection of units of product picked for the purpose of sampling. Based on the results of inspection of a random sample from the lot a decision is made to accept or reject all the units in the lot. The act of accepting or rejecting the entire lot is called* **sentencing the lot**.

The number of units in a lot is called the **lot size.** The number of units inspected to sentence a lot is called the **sample size.** The *proportion of defective items* in a lot is called the **lot quality**. Throughout, we shall use (i) **N** for the *lot size*, (ii) **n** for its *sample size*, and (iii) **p** for the *lot quality*.

There are three approaches to *lot sentencing*: (1) no inspection; (2) 100% inspection; and (3) acceptance sampling. Here, we have to discuss the acceptance sampling and, to understand this in a better way, let us start with a simple example of an acceptance sampling plan for Mr. Anil, which we denote by ASP1.

The maximum allowable number of defective units in a sample is called the acceptance number, which we denote by **c**.

ASP1. *From each lot of 1000 pens, take 100 at random and inspect them. Accept the lot if the inspected sample contains at most one defective pen; otherwise reject it.*

Thus, for ASP1, N = 1000 and n = 100. So, if a lot consists of 20 defective pens, then the lot quality p = 0.02. Also, observe that acceptance number c = 1 in this case.

Now identify these quantities in the following exexcise.

E10) Specify N, n and p for

(a) a lot of 400 bolts having 36 defective bolts;
(b) a box of 50 cricket balls having two defective balls;
(c) a situation when a sample of 40 bolts is drawn from a lot of 400 bolts and the lot is rejected if the sample contains two or more defectives.

For sentencing, suppose a lot is subjected to ASP1 by Mr. Anil. What do you think is the probability that the lot will be accepted? Of course, this is *one*, if all the pens in the lot are good; and it is *zero*, if all the pens in the lot are defective. Thus, the probability of accepting a lot, denoted by $\mathbf{P_a}$, depends on the number of defective items in the lot.

Suppose a lot has 20 defective items under ASP1. What is P_a for such a lot? Since *ASP1 allows at most one defective pen* in a sample of 100 pens, this probability is given by

$$P_a = P[X = 0] + P[X = 1],$$

where X is the number of defective pens in the sample.

We know that, in general, X has hypergeometric distribution and, so, we can compute the above probabilities using this fact. However, we know that when lot size N is large compared to sample size n, these probabilities can be closely approximated by assuming that X follows binomial distribution with parameters n and p.

Hypergeometric distribution with parameters N, G and n is the distribution of the number of *good* objects in a simple random sample of size n from a population of N objects of which G are *good*.

In fact, when $N \geq 10n$, it hardly makes any difference whether the probabilities are computed using *hypergeometric distribution* or *binomial approximation*. The advantage of using binomial approximation is that the formulae are simple and the numbers are small.

In general, the (binomial) *probability of observing exactly k defects in a sample of size n* is given by

$$P[X = k] = {}^nC_k\, p^k(1-p)^{n-k},$$

where ${}^nC_k$ stands for the number of ways of choosing k items out of n items. And, the probability of acceptance ($\mathbf{P_a}$) is given by

$$P_a = P[X \leq k] = \sum_{d=0}^{k} {}^nC_d\, p^d(1-p)^{n-d} \qquad (11)$$

Thus, for $n = 100$ and $p = 0.02$, we get $P_a = P[X \leq 1] = 0.1326 + 0.2706 = 0.4032$.

Note that this means that if a number of lots with $p = 0.02$ are subjected to ASP1, then only about 40.32% of them will be accepted and about 59.68% of the lots get rejected, even though their quality is same as those accepted.

Since P_a depends on p, from now onwards, we shall write it more explicitly as $P_a(p)$. Then, by above calculations, $P_a(0.02) = 0.4032$, for ASP1.

Try the following exercise.

E11) For ASP1, compute (a) $P_a(0.03)$; and (b) $P_a(0.05)$.

The three curves shown in Fig. 10 (for $c = 0, 1,$ and 2) are developed by evaluating Eqn.(11) for various values of p, $0 \leq p \leq 1$. So, each point on a curve is represented by $(p, P_a(p))$.

Each curve is an **oc curve** (*operating characteristic curve*) for some acceptance sampling plan. The preference of an acceptance sampling plan is completely described by its *oc curve*.

In view of Eqn.(11), a typical *oc curve* is a pictorial representation of the relationship between the *lot quality* (p) and the *probability of acceptance* ($\mathbf{P_a}$), for a given sampling plan. The greater the slope of an *oc curve*, the greater is the discriminatory power. However, as c decreases, the *oc curve* gets shifted to left (without much a change in its slope).

In general, the exact shape of a specific *oc curve* depends on the values of parameters

AQL, LTPD, α and β. Very shortly, we will define these four terms in this section.

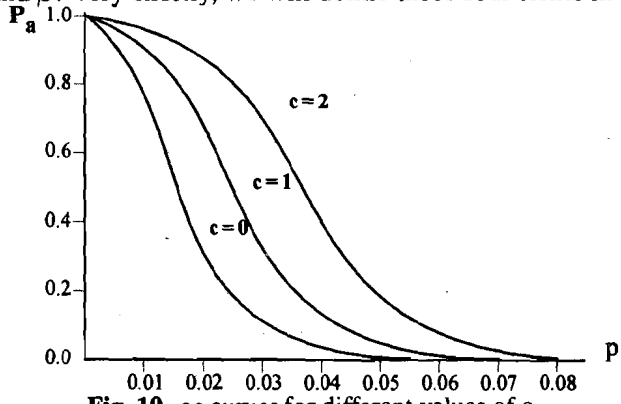


Fig. 10. *oc curves* for different values of c.

Here, we ask: What will happen if we change sampling plan ASP1? Let us consider another acceptance sampling plan with c = 3, which we denote by ASP2.

ASP2. *From each lot of 1000 pens, inspect 100 at random and accept the lot if the inspected sample has at most three defective pens.*

Once again, let us compute P_a for a lot which has exactly 20 defective items (i.e., p = 0.02). Under ASP2, it is given by

$$P_a(0.02) = \sum_{d=0}^{3} {}^{n}C_d p^d (1-p)^{n-d} = 0.86.$$

On comparing, we find that a lot of the *same quality* (as p = 0.02, in each case) has a chance of 0.40 (rounded off to two decimal points) of getting accepted, if it is subjected to ASP1, and has a chance of 0.86, if it is subjected to ASP2. So, we conclude that the probability of accepting a lot depends on both (a) the lot quality; and (b) the *acceptance sampling plan.*

Just as for ASP1, we can have *oc curve* for ASP2. In Fig. 11, the *oc curves* for three different acceptance plans are shown for comparison.

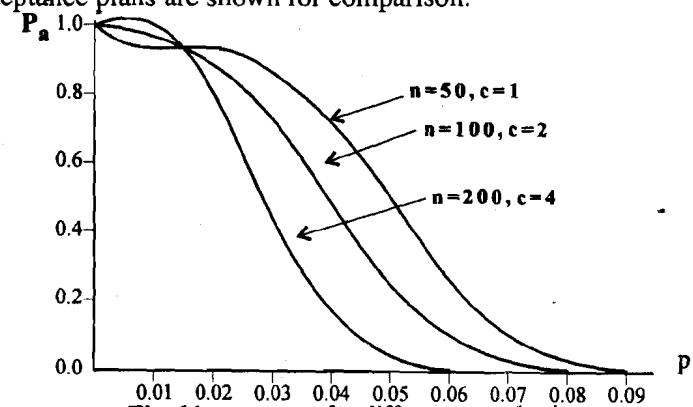


Fig. 11. *oc curves* for different sample sizes.

In Table 10, we have listed P_a values under ASP1 and ASP2 for some selected values of p. Examine these values to compare the two sampling plans.

Table 10 : Lot acceptance probabilities, P_a

p	P_a under ASP1	P_a under ASP2
0.000	1.0000	1.0000
0.010	0.7358	0.9816
0.020	0.4033	0.8590
0.030	0.1946	0.6472
0.046	0.0525	0.3196
0.076	0.0034	0.0490
0.100	0.0003	0.0078

We shall continue discussing *oc curve* a little later. Here, we take a break to talk about certain important parameters, which are used while adopting an acceptance sampling plan.

Both, M/s BP & Co. and Mr. Anil, understand that supply of completely defect-free lots is not possible. So, they have to come to a compromise. This is how a bargain starts between the two. Finally, they reach to an agreement : *Mr. Anil will accept majority of lots which have at most 1% defective pens (i.e.,* $p = 0.01$*) and the company will take back all those lots which contain more than 1% defective pens.*

Acceptable Quality Level

A level of quality, which is mutually agreed upon by both the buyer and the seller is called **acceptable quality level** (AQL). Thus, in view of above agreement between Mr. Anil and M/s BP & Co., AQL $= 0.01 (= p)$. This means that Mr. Anil should accept lots with $p \leq 0.01$ in *majority* of the cases.

Now, let us examine the consequence of Mr. Anil's decision, when AQL $= 0.01$ and ASP1 is adopted by him.

From Table 10, we find that $P_a(0.01) = 0.7358$. So, under ASP1, Mr. Anil will reject 26.42% (=100(1-0.7358)) of the lots whose quality is 0.01. This is obviously a *risk* to the producer because it was agreed upon by both that lots of quality 0.01 will be accepted in majority of the cases whereas Mr. Anil is rejecting 26.42% of them. Of course, a lot with $p < 0.01$ has a smaller chance of getting rejected than 26.42% (see Table 10). In other words, the producer's risk for any $p <$ AQL is less than 26.42%.

Producer's Risk

So, here is an important observation to note; *Among all values of* p *between* 0 *and* AQL, *producer's risk is maximum when* p = AQL. *Producer's risk* is defined as the probability of rejecting a lot whose p = AQL and is denoted by α. Usually it is expressed as a percentage. So, producer's risk in above situation is given by $\alpha = 100(1 - P_a(AQL))\%$.

What will happen if Mr. Anil adopts ASP2 instead of ASP1? Try the following exercise to find the answer.

E12) Assuming ASP2 is adopted, find out $P_a(0.01)$. Also locate (roughly) the points AQL(= 0.01) on x-axis, producer's risk α on y-axis and (AQL, P_a(AQL)) on the two *oc curves* plotted under ASP1 and ASP2.

With AQL $= 0.01$ in above exercise, you must have got the producer's risk as 1.84% under ASP2. Thus, under ASP1, the producer's risk is more and, so, the company would say to Anil: *26.42% is too much of a risk for us, we would bear at most 5% risk.* What is the solution? We can change the sampling plan.

On the other hand, under ASP2, the producer's risk is only 1.84% (much less than the desired 5%). So, the company will be complacent. But then, what will happen to Mr. Anil as a consumer?

From Eqn.(11), we find that $P_a(0.05) = 0.2578$, under ASP2. This means that, if ASP2 is adopted, then lots whose quality is as bad as having 5% defective pens (this is much worse than the agreed upon quality AQL $= 0.01$) will get accepted 25.78% of the time. Obviously, this is a risk to the consumer.

Consumer's Risk

Why should Mr. Anil accept such bad lots 25.78% of the time? Accepting lots whose quality is worse than AQL, the consumer is at a loss. Thus, ASP2 is not good for Mr. Anil even though it is good for the producer.

If we want a sampling plan that is best for both the consumer and the producer, then all those lots with $p \leq$ AQL should be accepted with probability 1 and all those lots with $p >$ QAL should be rejected with probability 1 (or accepted with probability 0).

The *oc curve* of such a sampling plan will look like the one in Fig. 12 and is called an **ideal oc curve**. It is clear that such an acceptance sampling plan would call for almost

100% inspection and the inspection costs will be high.

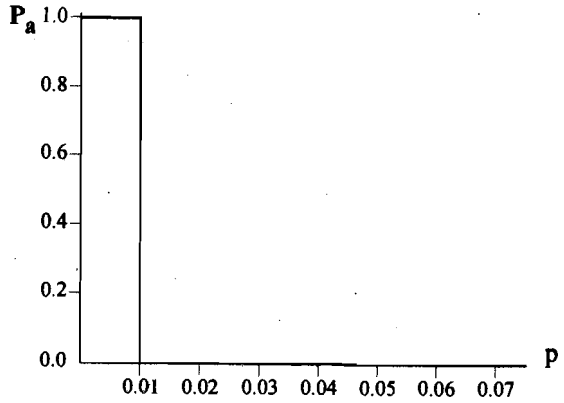


Fig. 12. *Ideal oc curve.*

Lot Tolerance Proportion Defective

As a compromise, Mr. Anil is ready to *tolerate* lot qualities worse than AQL upto certain limit, but not beyond. Let us assume Mr. Anil is willing to tolerate lots with $p \geq 0.05$ but not not more than 10% of the time. The *tolerance limit* on the lot quality $p = 0.05$ that he has chosen to tolerate, is called the **lot tolerance proportion defective** and is denoted by **LTPD**. Thus, LTPD = 0.05 in case of Mr. Anil.

In the above paragraph, we have mentioned that Mr. Anil is willing to accept lots with quality p = LTPD only 10% of the time. Here, 10% is what we call the consumer's risk.

The probability of accepting a lot with p = LTPD *is called the consumer's risk* and is denoted by β. In other words, *consumer's risk* is equal to P_a(LTPD). Like the producer's risk, the consumer's risk is also usually expressed as a percentage.

Above we have seen that while ASP1 is good for the consumer, ASP2 is good for the producer. But neither of the two plans will satisfy both of them. So, we should look out for a sampling plan that should be acceptable to both consumer and producer.

It is customery to use the AQL and LTPD points for this purpose and the corresponding points on the *oc curve*, α and β, respectively, give producer's and consumer's risk (see Fig. 13). Such an acceptance sampling plan protects the interest of both producer and consumer.

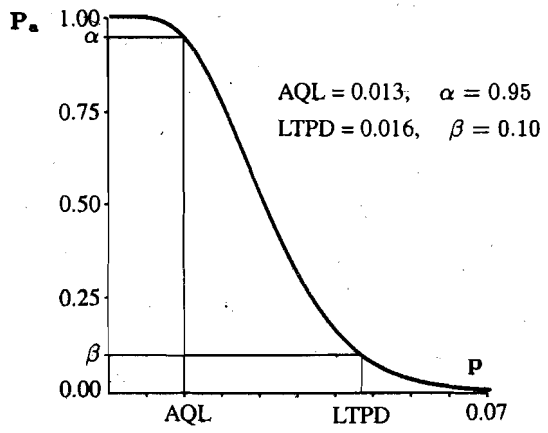


Fig. 13

Note that P_a for any lot with $p < AQL$ is at least $1 - \alpha$ (or probability of rejecting such a lot is at most α) and P_a for any lot with $p > LTPD$ is at most β.

If you want to have cake and eat it too, then you must have too many cakes.

Obtaining acceptance sampling plans that protect both producer and consumer usually calls for large sample sizes. So, in practice, one has to compromise somewhere.

A sampling plan never guarentees the acceptance of 100% perfect material. In fact, some defective material will also get accepted in the process. But, by proper specification of β risk, it is possible to minimise the amount of defective material.

Thus, a sampling plans is used to have a reasonably good idea about how much unacceptable material will be involved.

11.4.2 Single Sampling Plans

We use some statistical and probability tools to develop sampling plans that meet the desired α & β risks and maintain the desired AQL and LTPD quality levels. Of course, our main objective is to determine P_a of lots with varying quality.

There are many types of acceptance sampling plans. As before, we will confine our discussion to *sigle sampling plans* and learn how to design them. Other types of sampling plans will be discussed very briefly at the end of the section.

Single Sampling Plans

Already, we discussed some examples like *ASP1* and *ASP2*. More generally, a *single sampling plan* is completely described by specifying (i) the lot size, **N**; (ii) sample size, **n**; and (iii) acceptance number, **c**.

In a *single sampling plan*, given by the three values (N, n, c), we inspect n items at random from a lot of N items and accept the lot if the number of defective items in the sample is less than or equal to the acceptance number c; otherwise we reject the lot. And, throughout, we use *simple random sampling without replacement* for sampling items.

A simple graphical procedure, called a *binomial nomograph*, is used to construct single sampling plans for a specified AQL, LTPD, α and β (see Fig. 14).

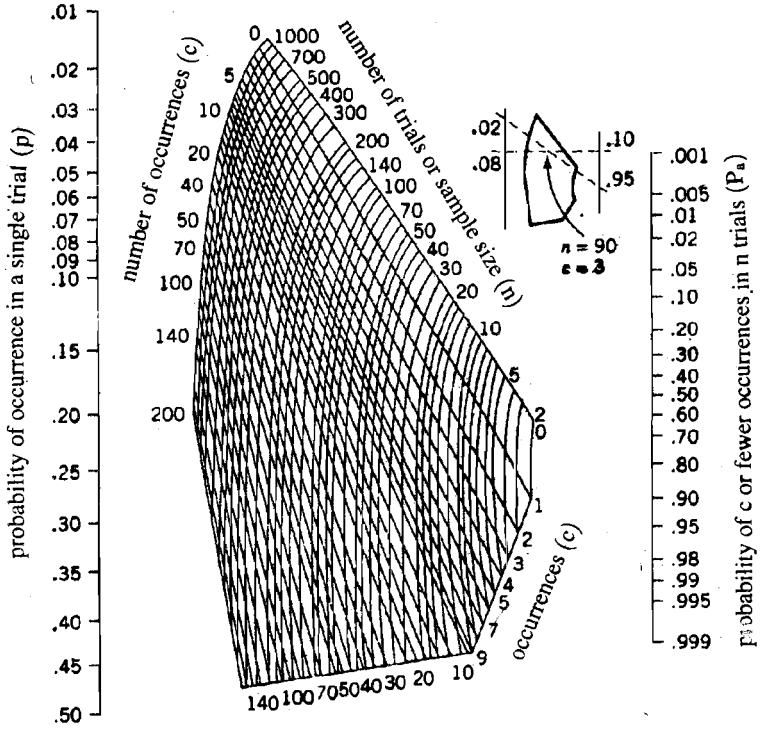


Fig. 14. (*Source*: D.C.Montgomery, Introduction to Statistical Quality Control (4/e),2001.)

As an illustartion, we try to find a single sampling plan when AQL = 0.02, LTPD = 0.08, $\alpha = 0.05$ and $\beta = 0.10$.

Firstly, draw a straight line joining 0.02 on the p-scale and 0.95 ($= 1 - \alpha$) on the P_a-scale. Draw another straight line joining 0.08 on the p-scale and 0.1($= \beta$) on the P_a-scale. Now, read the values of n and c corresponding to the point of intersection of the two lines drawn. You can see that we will get n = 90 and c = 3. Observe that we have not talked about lot size anywhere in the process.

Infact, the lot size is implicitly assumed to be at least 10 times the sample size. Thus, if

Mr. Anil wants to use the above derived plan, his lot size should be at least 900. And, as N = 1000 for refill length problem, the above plan can be used.

Try the following exercise.

E13) From the nomograph shown in Fig. 14, derive single sampling plans when
(i) AQL = 0.01, LTPD = 0.10, $\alpha = 0.05$, $\beta = 0.10$; and
(ii) AQL = 0.03, LTPD = 0.08, $\alpha = 0.05$, $\beta = 0.10$.

Percentage Sampling

In the past, it was a common practice in industry to inspect certain percentage of items in the lots. In other words, single sampling plans of the type (100, 10, 0), (500, 50, 0) and (1000, 100, 0) were often being used. The sample size in all these plans is 10% of the lot size.

This may make us believe that all these plans are equally good. No, they are not! On examining the *oc curves* for the 3 plans mentioned in this paragraph with c = 0 (see Fig. 15), it is sufficiently clearly that the three plans are drastically different.

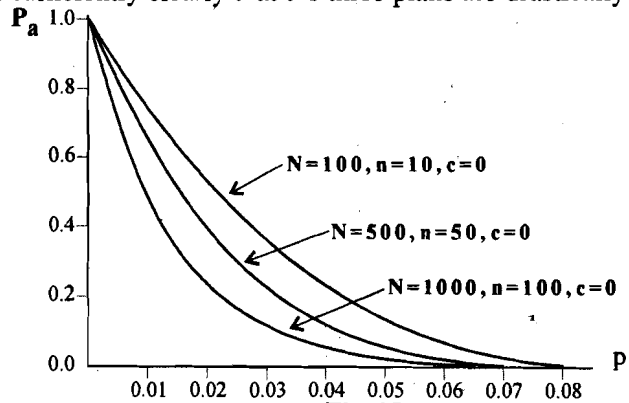


Fig. 15

The main disadvantage in this approach is that *different sample sizes offer different levels of protection.* As such, it is illogical for the level of protection the consumer enjoys for a critical part to vary as the size of the lot varies.

You will find the following exercise more convincing.

E14) Assuming AQL = 0.05, read, roughly, the producer's risks for the 3 plans in Fig. 15.

There are many other sampling plans such as *double sampling plans, multiple sampling plans, sequential sampling plans, chain sampling plans*, and so on. A number of published sampling plans are developed from various view points. Of course, many computer softwares are available today in the market with which you can desing and evaluate various sampling plans and schemes at your finger tips.

With this we have come to the end of the block. Let us summarise what we have learnt in this unit.

11.5 SUMMARY

In this unit, we have discussed with you the following aspects of SQC.

1. The concept of quality and its characteristics.
2. How quality of a product is achieved by ensuring that quality characteristics conform to their specifications.
3. Use of control charts as a primary tool for an on-line process control.
4. The construction and application of $\bar{x}$ - R charts for controlling variable characteristics.

5. The meaning of process capability and its evaluation through $\bar{x}$ - R charts.
6. The construction and application of **p**, **np**, and **c** charts for controlling attribute quality characteristics.
7. The concept of acceptance sampling plans and use of the oc curve, AQL, producer's risk, consumer's risk, and LTPD, while arriving at a plan that protect the interest of both consumer and producer's equally. Also, we have seen how oc curves have been used in comparing sampling plans.
8. The construction of a single sampling plan using *binomial nomograph.*

11.6 SOLUTION/ANSWERS

E1) Measurable characteristics such as diameter of a ball, length of refill, weight of cricket ball, thickness of a washer etc., are suitable for control charts for variables. Characteristics such as defective ball pens, defects such as scratches on a cricket ball, neps and faded portions on a piece of cloth, etc., are suitable for control charts for attributes.

E2) Estimate of process mean is given by

$$\bar{\bar{x}} = \frac{\text{sum of sample averages}}{\text{number of samples}} = \frac{7575}{25} = 303 \text{ grams.}$$

Estimate of process standard deviation is given by

$$\frac{\bar{R}}{d_2} = \frac{\text{sum of sample ranges/number of samples}}{2.059} = \frac{154/25}{2.059} = 2.992 \text{ grams.}$$

Then, the *control limits for* $\bar{x}$ - *chart* are given by

CL=303, $LCL = \bar{\bar{x}} - A_2\bar{R} = 303 - 0.73 \times 6.16 = 298.503$,

$UCL = 303 + 0.73 \times 6.16 = 307.497$.

And, the *control limits for* R-*chart* are given by

$CL = \bar{R} = 6.16$, $LCL = d_3\bar{R} = 0$, $UCL = d_4\bar{R} = 2.28 \times 6.16 = 14.045$.

E3) After dropping 12th subgroup,

$$\bar{\bar{x}} = \frac{201.808 - 10.086}{19} = 10.091, \ \bar{R} = \frac{3.51 - 0.38}{19} = 0.1647.$$

Therefore the revised control limits, for $\bar{x}$ - chart, are

$LCL = 9.995$, $CL = 10.091$, $UCL = 10.186$; and

$LCL = 0$ $CL = 0.1647$, $UCL = 0.3476$, for R - chart.

From Table 1, we find that none of the points (excluding 12th subgroup), either on $\bar{x}$ - chart or R - chart, fall outside control limits. So, the remaining data indicate statistical control.

E4) We may take refill length problem. Here, total tolerance (= T) = 0.4. Then,

$$C_p = \frac{0.4}{6\sigma} = 1.5 \ \Rightarrow \ \sigma = \frac{1.6}{36} = 0.044.$$

Thus, (a) rejctions will occur on USL side when $\mu + 3\sigma >$ USL. For instance, if $\mu = 10.111$ (i.e., $\mu + 2\sigma =$ USL), there will be rejections on USL side. And, (b) rejctions will occur on LSL side when $\mu + 3\sigma <$ LSL. For instance, if $\mu = 9.889$ (i.e., $\mu - 2\sigma =$ LSL), there will be rejections on LSL side.

E5) For the combined data, $\bar{\bar{x}} = \frac{200.177}{20} = 10.009$ and $\bar{R} = \frac{2.65}{20} = 0.1325$. Thus,

(a) estimates of mean and standard deviation for the new process are given by

$$\hat{\mu} = \bar{\bar{x}} = 10.009 \text{ and } \hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{0.1325}{2.326} = 0.0569.$$

And, (b) the control limits, for $\bar{x}$ - chart, are given by

$LCL = 9.932$, $CL = 10.009$, $UCL = 10.086$; and

$LCL = 0$, $CL = 0.1325$, $UCL = 0.2795$, for R - chart.

Again, (c) the first subgroup average is below LCL. on $\bar{x}$ - chart. Possibly there was an assignable cause. But for this, the data indicate statistical control. And, (d) since R-chart indicates control, we might use all the 20 subgroups to estimate the process variability. An estimate of the process capability is given by

$$6\hat{\sigma} = 6 \times \frac{\bar{R}}{d_2} = 6 \times \frac{0.1325}{2.326} = 0.3414,$$

and, an estimate of process capability ratio is given by

$$\hat{C}_p = \frac{T}{6\hat{\sigma}} = \frac{0.4}{0.3414} = 1.171.$$

Finally, (e) since $\hat{C}_p < 1.33$, further reduction in process variability is essential. So, the advice is that the manager should explore the possibilities of reducing variation further.

E6) (b) There is clear indication that the process average has shifted downwards. (c) We cannot comment on the process control unless we redraw the control limits. (d) For the data in Table 6, $CL = \bar{p} = \frac{19}{10 \times 100} = 0.019$,

$LCL = 0.019 - 3\sqrt{\frac{0.019 \times 0.981}{100}} = -0.02$, $UCL = 0.019 + 3\sqrt{\frac{0.019 \times 0.981}{100}} = 0.06$. Since, $LCL < 0$, LCL is plotted at 0.0. (e) Data indicate improvement in the process. The rejection percentage for the first period is equal to 6% (= 100 × old $\bar{p}$ estimate) and for the latter period it is equal to 1.9% (=100 × new $\bar{p}$ estimate).

E7) The estimate of process average is given by $\bar{p} = \frac{7}{253} = 0.013$. In this case, the LCL turns out be zero for all the five subgroups. The UCL for subgroups with subgroup size 100 is equal to $0.013 + 3\sqrt{\frac{0.013 \times 0.987}{100}} = 0.048$. The table below summarises the UCLs and sample averages for the data.

S.No.	1	2	3	4	5
sample size	100	121	81	100	121
sample average	0.020	0.016	0.000	0.010	0.016
UCL	0.048	0.045	0.052	0.048	0.045

E8) YES. The control limits will vary with varying sample sizes. Even the center line will vary because $CL = n\bar{p}$.

E9) (a) Estimated number of defects per carton = $\frac{150}{15} = 10$. (b) The control limits are : $CL = 10$, $LCL = 10 - 3\sqrt{10} = 0.513$ and $UCL = 19.487$. Since what we plot on the y-axis is the total number of defects per carton, we may take $LCL = 1$ and $UCL = 19$. Clearly, assembly process of Group E is under statistical control. (c) Since the average number of defects per carton of Group A is only 4.8, Group A is better. (d) If the chart is displayed in front of the operators, they will have a continuous feed back on their performance. So, whenever the quality deteriorates, they can correct themselves. Psychologically, it will have good impact on the operators.

E10) (a) $N = 400, p = 0.09$; (b) $N = 50, p = 0.04$; (c) $N = 400, n = 40, p = 0.005$.

E11) (a) $P_a(0.03) = 0.1946$; (b) $P_a(0.05) = 0.0371$.

E12) Under ASP2, $P_a(0.01) = 0.9816$ (see Table 10). The actual points are $(0.01, 0.7358)$, for ASP1, and $(0.01, 0.9816)$, for ASP2. So, these should be the points that you read from the graph approximately.

E13) The single sampling plans from the nomograph are (i) (N, 40, 1) and (ii) (N, 200, 10). Here, the lot size N should be at least 10 times the the corresponding sample size.

E14) The producer's risks are: 0.2141, for (1000,100,0); 0.2578, for (500,50,0); and 0.1498, for (100,10,0).

APPENDIX

TABLE-1
F-distribution

$F_{0.05}$

ν_2 = Degrees of freedom for denominator	ν_1 = Degrees of freedom for numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.50	19.00	19.20	19.20	19.30	19.30	19.40	19.40	19.40	19.40	19.40	19.40	19.40	19.50	19.50	19.50	19.50	19.50	19.50
3	10.10	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.38	2.30	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	3.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.93
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

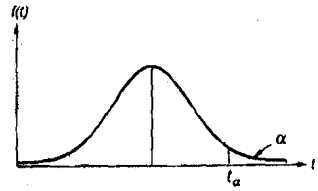
TABLE-1(Continued)
F-distribution

$F_{0.01}$

ν_2 = Degrees of freedom for denominator	ν_1 = Degrees of freedom for numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,982	6,023	6,056	6,106	6,157	6,209	6,235	6,261	6,287	6,313	6,339	6,366
2	98.50	99.00	99.20	99.20	99.30	99.30	99.40	99.40	99.40	99.40	99.40	99.40	99.40	99.50	99.50	99.50	99.50	99.50	99.50
3	34.10	30.80	29.50	28.70	28.20	27.90	27.70	27.50	27.30	27.20	27.10	26.90	26.70	26.60	26.50	26.40	26.30	26.20	26.10
4	21.20	18.00	16.70	16.00	15.50	15.20	15.00	14.80	14.70	14.50	14.40	14.20	14.00	13.90	13.80	13.70	13.70	13.60	13.50
5	16.30	13.30	12.10	11.40	11.00	10.70	10.50	10.30	10.20	10.10	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.70	10.90	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.20	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.30	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.83
9	10.60	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.00	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.96	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.53	2.45	2.36	2.27	2.17
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

TABLE-2

t-distribution



ν	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$	$t_{.0005}$
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291